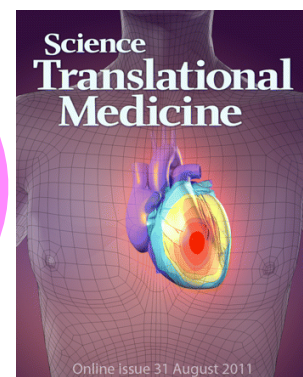
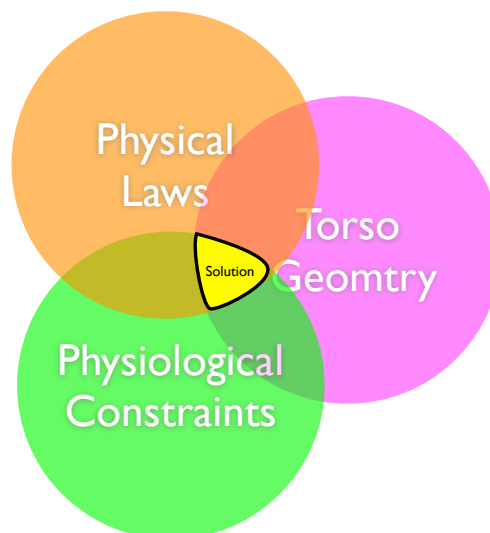
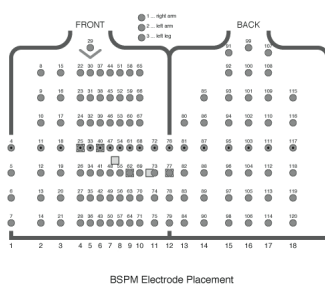


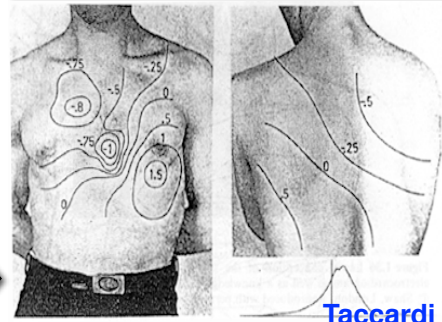
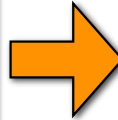
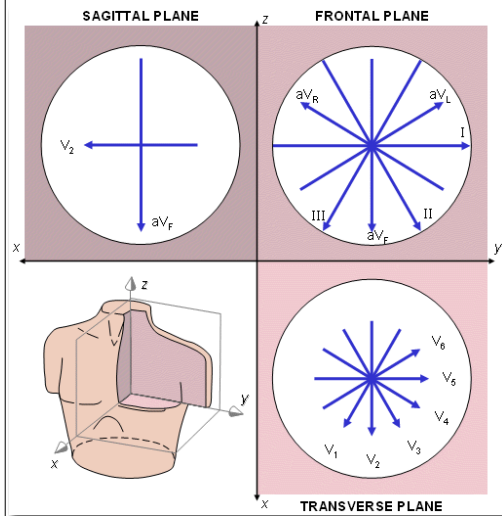
# Inverse Problems in Electrocardiography

## What is ECGI?

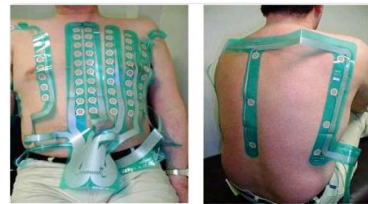




# ECG to BSPM



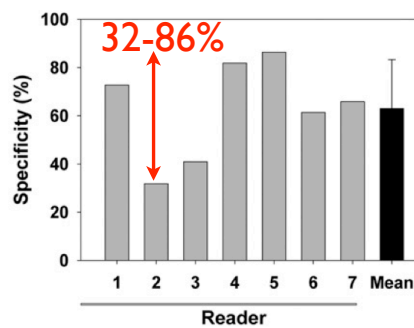
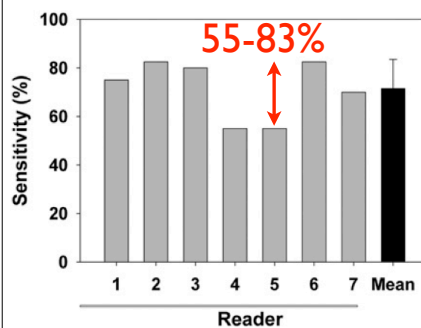
Taccardi et al,  
Circ., 1963



## Performance of ECG

### Differentiating ST-Elevation Myocardial Infarction from Nonischemic ST-Elevation in Patients With Chest Pain

Viet Tran, MD<sup>a</sup>, Henry D. Huang, MD<sup>a</sup>, Jose G. Diez, MD<sup>a,b</sup>, Gerardo Kalife, MD<sup>b</sup>, Rajiv Goswami, MD<sup>a</sup>, David Paniagua, MD<sup>a</sup>, Hani Jneid, MD<sup>a</sup>, James M. Wilson, MD<sup>a,b</sup>, Scott R. Sherron, MD<sup>b</sup>, and Yochai Birnbaum, MD<sup>a,b,\*</sup>



J Cardiol 2011;108:1096-1101



# More Leads = More Information

## Mechanisms of the Spatial Distribution of QT Intervals on the Epicardial and Body Surfaces

BONNIE B. PUNSKE, Ph.D., ROBERT L. LUX, Ph.D.,  
ROBERT S. MACLEOD, Ph.D., MARC S. FULLER, Ph.D.,  
PHILIP R. ERSHLER, Ph.D., THEODORE J. DUSTMAN, M.E.,  
YONILD VYHMEISTER, M.S., and BRUNO TACCARDI, MD, Ph.D.

*J. Cardiovasc. Electrophys.* Volume 9, Issue 7, pages 773–786, July 1998

*Adv Cardiol.* 1978;21:36-9.

**Criteria for localizing preexcited areas from body surface maps in Wolff-Parkinson-White patients.**

De Ambroggi L, Taccardi B, Macchi E, Perotta GM.

**Body surface mapping during percutaneous transluminal coronary angioplasty. QRS changes indicating regional myocardial conduction delay**

H Spekhorst, A SippensGroenewegen, GK David, MJ Janse and AJ Dunning

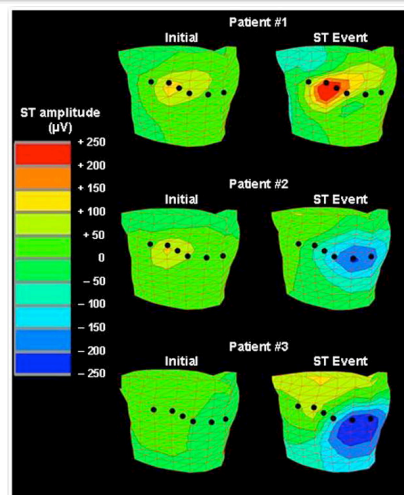
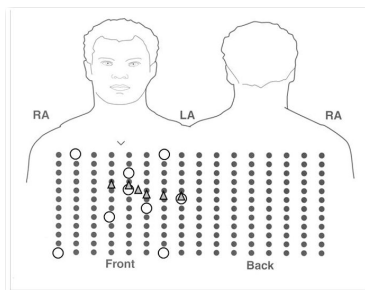
*Circulation* 1990, 81:840-849



## Example

Estimated body surface potential maps in emergency department patients with unrecognized transient myocardial ischemia<sup>☆</sup>

Barbara J. Drew, RN, PhD,<sup>a,b,\*</sup> Daniel M. Schindler, RN, MS,<sup>a</sup> Jessica K. Zegre, RN, MS,<sup>a</sup>  
Kirsten E. Fleischmann, MD,<sup>b</sup> Robert L. Lux, PhD<sup>c</sup>

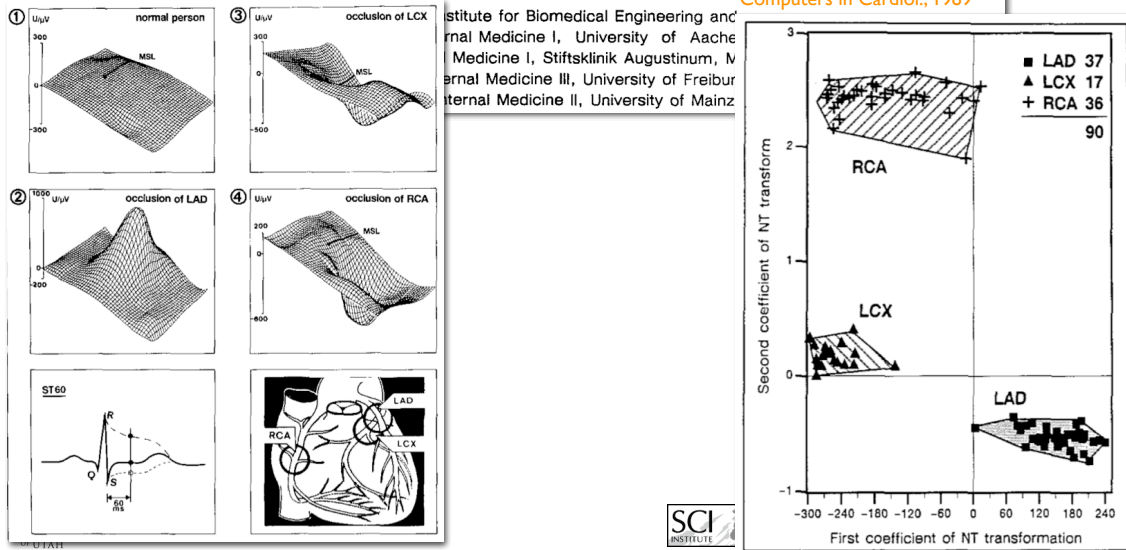


# PTCA Example

## VALIDATION OF BODY SURFACE POTENTIAL MAPPING DURING THROMBOLYSIS: MULTI-CENTRE STUDY BY MEANS OF A COMMUNICATION COMPUTER NETWORK

L. Vogt, J. Silny, G. Rau, S. Effert,  
R. Uebis<sup>1</sup>, R. von Essen<sup>2</sup>, H. Just<sup>3</sup>, J. Meyer<sup>4</sup>

Computers in Cardiol., 1989



# How Much Information?

## How Many Leads Are Necessary for a Reliable Reconstruction of Surface Potentials During Atrial Fibrillation?

María de la Salud Guillem, Andreas Bollmann, Andreu M. Climent, Daniela Husser,  
Jose Millet-Roig, and Francisco Castells

IEEE Transactions on Information Technology in Biomedicine, 2009, 13, 3, 330

## Standardization of reduced and optimal lead sets for continuous electrocardiogram monitoring: where do we stand?

Barbara J. Drew, PhD,<sup>a,\*</sup> Dewar D. Finlay, PhD<sup>b</sup>

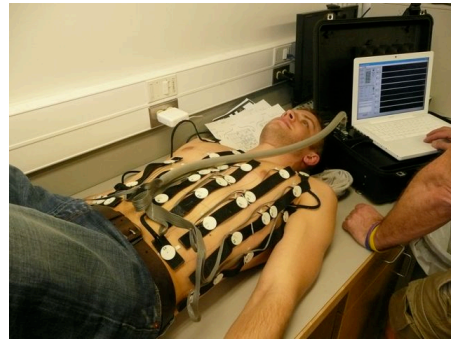
<sup>a</sup>Department of Physiological Nursing, University of California San Francisco, San Francisco, CA, USA

<sup>b</sup>School of Computing and Mathematics and Computer Science Research Institute, University of Ulster, Northern Ireland, UK

J Electrocardiol. 2008;41(6):458–465.



# More Leads = More Time



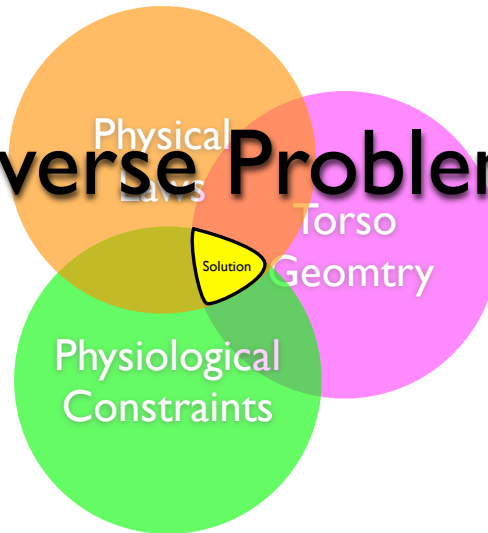
## BSPM Summary

- “...reliability and test performance of body surface mapping in CAD is promising”
- “The limited evidence that is available demonstrates proof of concept...”
- “Further research is needed to better characterize the performance characteristics of these devices...”

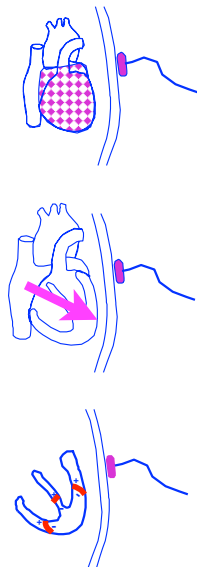
Aetna Clinical Policy Bulletin: Body Surface Potential Mapping, 2010.



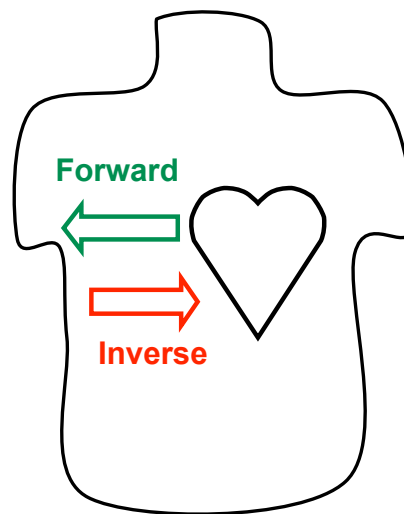
# Inverse Problem



## Bioelectric Field Problem Basics

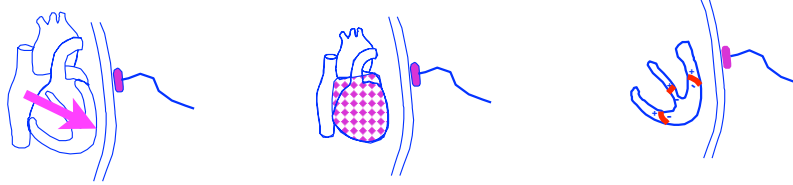


Source



Volume Conductor Model

# Sources



Dipole	Epicardial Potentials	Epi-Endocardial Activation Time
Simple, few leads, conventional	Measurable, comprehensive, unique	Measurable, clinically directly useful
Not unique, not measurable, requires assumptions, misses details	Interpretation ambiguous, complex, ill-posed	Uniqueness unclear, tenuous assumptions, ill-posed

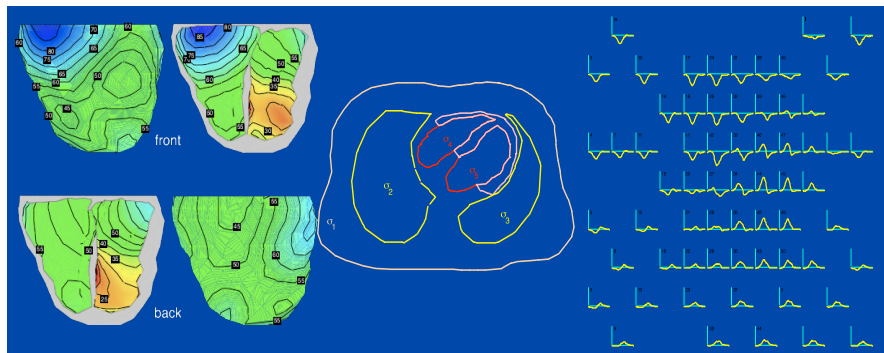
# Activation

Forward problem

Epicardial/Endocardial Activation Time

Geometric Model

Body Surface Potentials

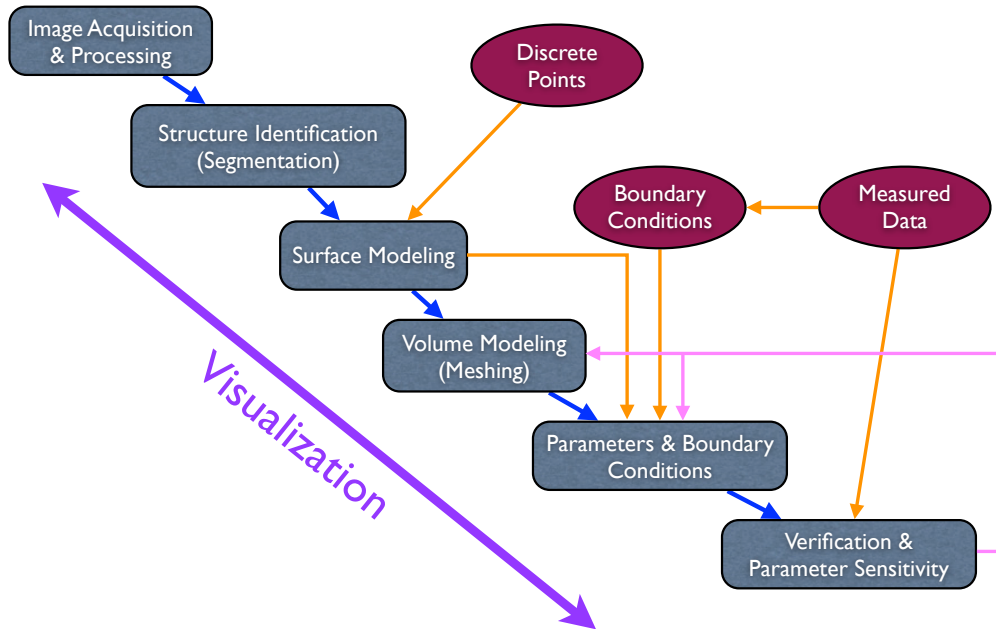


Inverse problem

Thom Oostendorp,  
Univ. of Nijmegen



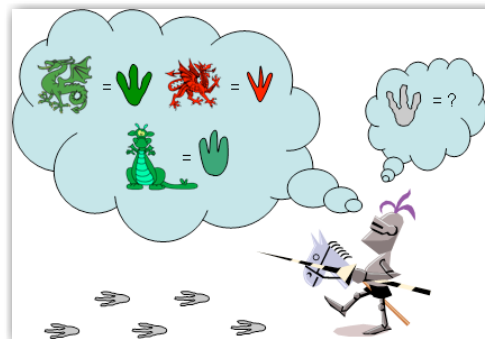
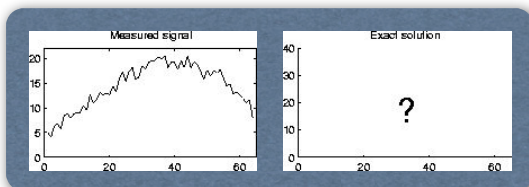
# How To Solve an Inverse Problem



# What Does Ill-Posed Mean?



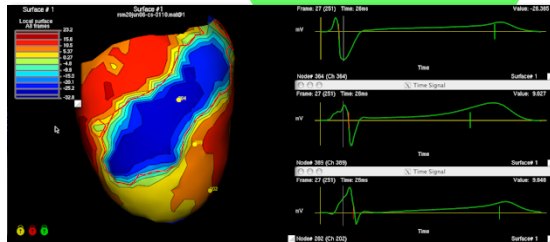
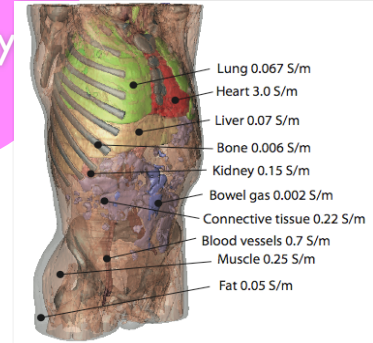
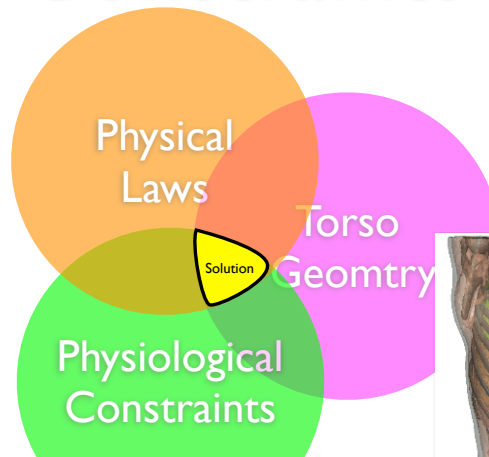
- ✓ A solution exists
- ✓ The solution is unique
- ✓ The solution depends continuously on the data, in some reasonable topology.



# Constraints

$$\nabla^2 \phi = -\frac{I_v}{\sigma}$$

$$\nabla^2 \phi = 0$$



# Applying Constraints

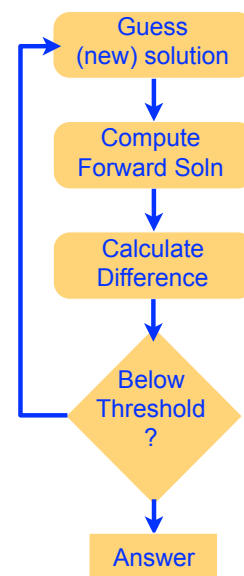
## Closed Form

$$x_{\lambda} = \arg \min_x (||y - Ax||^2 + \lambda^2 ||Mx||^2)$$

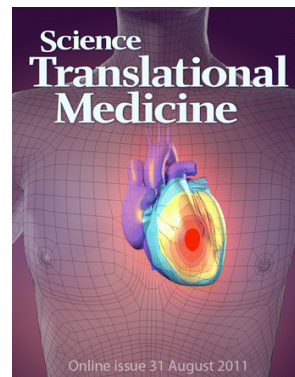
with solution

$$x_{\lambda} = (A^T A + \lambda M)^{-1} A^T y$$

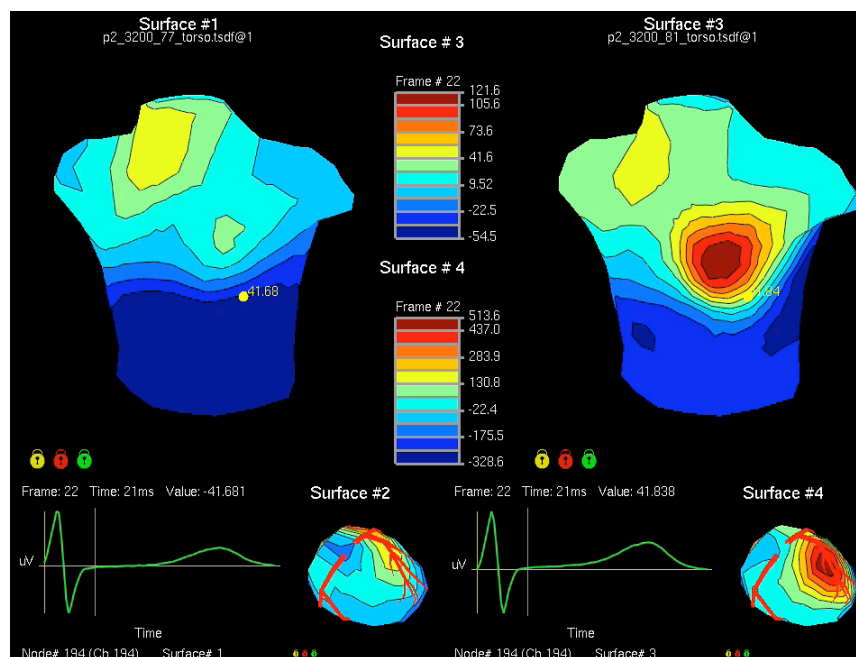
## Iterative



# Results



## Acute Ischemia



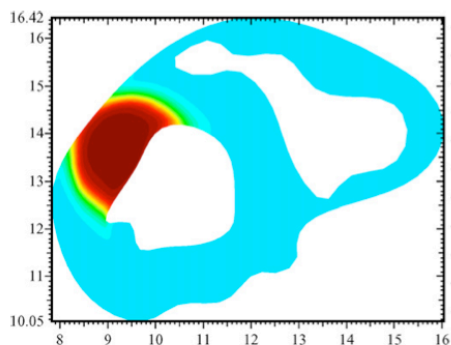
# On the possibility for computing the transmembrane potential in the heart with a one shot method: An inverse problem

Bjørn Fredrik Nielsen <sup>a,b,\*</sup>, Xing Cai <sup>a,b</sup>, Marius Lysaker <sup>a</sup>

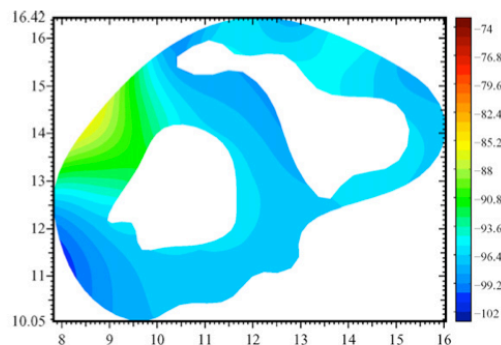
<sup>a</sup> Simula Research Laboratory, P.O. Box 134, N-1325 Lysaker, Norway

<sup>b</sup> Department of Informatics, University of Oslo, P.O. Box 1080, Blindern, N-0316 Oslo, Norway

Mathematical Biosciences. 2007;210(2):523–553.



(a)  $v_{\text{true}}$



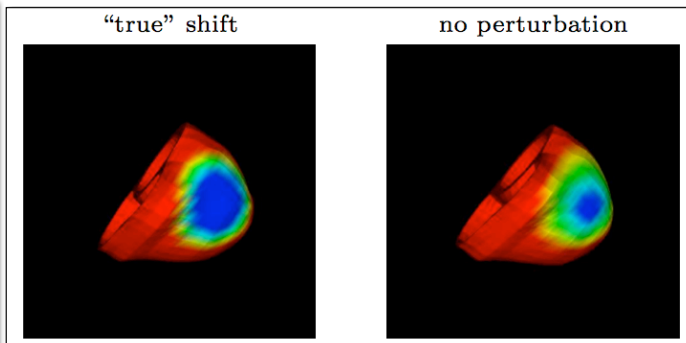
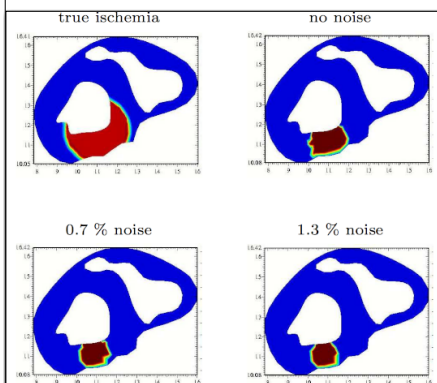
(b) Solution  $v_{\text{oneshot}}$



## CAN ECG RECORDINGS AND MATHEMATICS TELL THE CONDITION OF YOUR HEART?

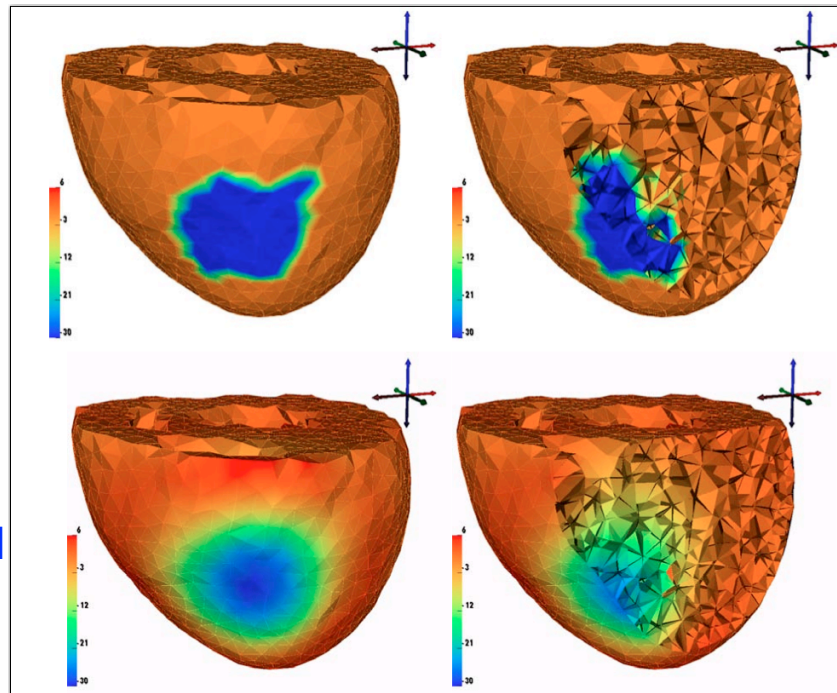
Bjørn Fredrik Nielsen, Marius Lysaker, Per Grøttum, Kent-André Mardal, Aslak Tveito, Christian Tarrø, Kristina Hermann Haugaa, Andreas Abildgaard, and Jan Gunnar Fjeld

SIMULA RESEARCH LABORATORY  
2010, Part 2, 287-319



True

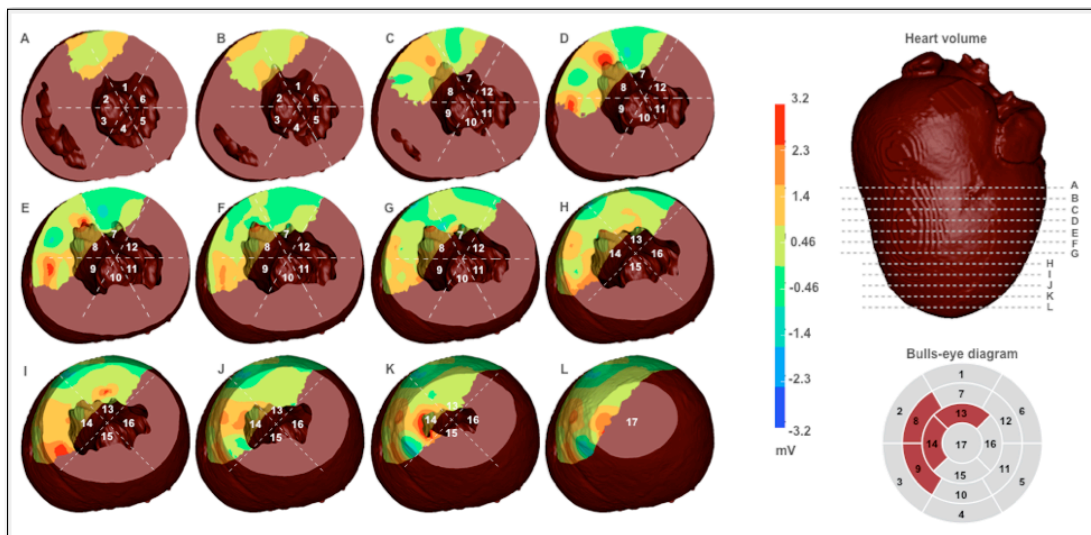
Inverse  
Computed



Dafang, Kirby, Johnson, Macleod, Unpublished Results



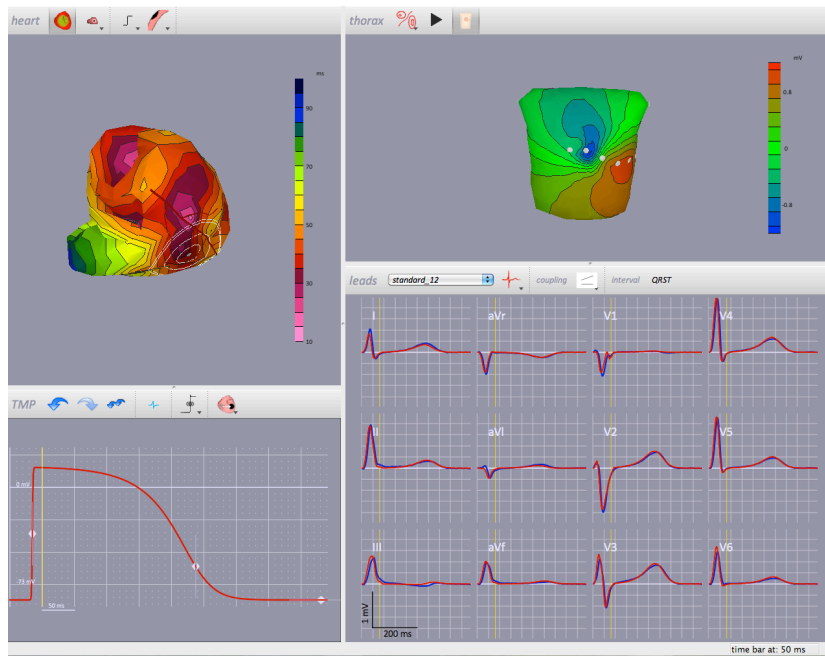
# Measurements



Aras, Swenson, Burton, Macleod, Unpublished Results



# Tools for Learning



ECGSim