

Statistical Uncertainty in the Medicare Shared Savings Program

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Technical Appendix:

Statistical Model of ACO Payment Rules in the Medicare Shared Savings Program

This technical appendix describes in detail the methodology we use to assess the statistical properties of the ACO payment mechanisms in the Medicare Shared Savings Program (MSSP). As described in the main text, the ACO savings rate (ASR) can be written as:

$$ASR = [(\bar{Y}_B + A) - \bar{Y}_P] / (\bar{Y}_B + A) \quad (A1)$$

where \bar{Y}_B is the ACO's risk-adjusted baseline per capita spending level (i.e., 3-year weighted average), \bar{Y}_P is the ACO's risk-adjusted performance year per capita spending level, and A is the projected absolute amount of growth in per capita Medicare expenditures nationally, introduced in the main text. In the one-sided model, the ACO is rewarded if $ASR > T_n$ where T_n is the MSR threshold set by CMS for an ACO with n assignees. (The method for determining T_n is described below.) In the two-sided model, an ACO would be rewarded if $ASR > 0.02$ and would pay a penalty if $ASR < -0.02$.¹

Sources of random variability in the payment formula

Our ultimate goal is to analyze the likelihood that an ACO would meet the relevant MSR threshold under alternative scenarios regarding its underlying true performance in controlling healthcare spending. We also wish to estimate the size of the expected ACO payment assuming the ACO has met the MSR threshold in each scenario. This analysis requires knowledge about random variability in ASR , which is a complicated function of the random variables \bar{Y}_B , \bar{Y}_P , and A .

¹This approach follows the general framework originally proposed by Fisher et al. (2009).

Variability in A is driven by factors affecting the growth in Medicare spending nationally, such as progression of illness among Medicare beneficiaries and changes in medical technology. Variability in \bar{Y}_B and \bar{Y}_P are driven by a number of additional random factors at the patient and ACO levels (including factors affecting the weighted components used to construct \bar{Y}_B). To understand this variability, we use the following variance components model of ACO spending:

$$Y_{it} = \mu_t + \varepsilon_{it} \quad (\text{A2})$$

where Y_{it} is risk and growth trend adjusted spending (as described in the main text) for ACO patient i at time t , and μ_t is the mean ACO spending per patient at time t , where $t=1, 2, 3$ are the baseline years and $t=4$ is the first performance year. If the ACO is effective at reducing growth trend/risk-adjusted healthcare spending, then μ_4 would be substantially less than μ_1, μ_2 , and μ_3 . The term ε_{it} is a random deviation, or error term, for patient i at time t . Deviations can be specific to the ACO, individual patients, or time. Thus, we model ε_{it} as

$$\varepsilon_{it} = u_t + v_{it} \quad (\text{A3})$$

where u_t is the random deviation in mean growth trend/casemix-adjusted healthcare spending at time t (experienced by all ACO patients) and v_{it} is the corresponding deviation that is specific to patient i at time t . Estimation and inference regarding μ_t requires assumptions about the components of ε_{it} . Following common practice in variance components and hierarchical linear modeling (Diggle, Heagerty, Liang, & Zeger, 2002), we assume that $E(u_t) = E(v_{it}) = 0$, $V(u_t) = \sigma_u^2$, and $V(v_{it}) = \sigma_v^2$. Although we will need to modify them later, a fully specified model would typically include the following additional assumptions:

1. The ACO-specific (u_t) and patient-specific (v_{it}) error components are uncorrelated.
2. Spending deviation for one patient is unrelated to spending deviation by other patients—i.e., $cov(v_{it}, v_{jt}) = 0$.
3. For any particular patient, spending may be correlated from one time period to the next—i.e., $cov(v_{is}, v_{it}) = \gamma$.
4. Random deviations in mean ACO health spending may be correlated over time—i.e., $cov(u_s, u_t) = \delta$.

Substituting equations 2 and 3 into equation 1 and accounting for the 3-year baseline calculation gives

$$ASR = 1 - [\mu_4 + \sum_{i=1}^n \varepsilon_{i4}/n] / [(\omega + A) + \tau] \quad (\text{A4})$$

where $\omega = 0.1\mu_1 + 0.3\mu_2 + 0.6\mu_3$ and $\tau = 0.1 \sum_{i=1}^n \varepsilon_{i1}/n + 0.3 \sum_{i=1}^n \varepsilon_{i2}/n + 0.6 \sum_{i=1}^n \varepsilon_{i3}/n$.

To understand how ACOs would be rewarded or penalized under the proposed rules, we need to calculate the probability that *ASR* crosses the relevant *MSR* threshold given a specified level of ACO savings. To do so, we define *s* as the proportion by which the ACO reduces average healthcare expenditure relative to $(\omega + A)$, which is the weighted average of mean ACO spending per patient in the baseline period plus the absolute change in per capita Medicare spending—i.e., $\mu_4 = (1 - s)(\omega + A)$. In the one-sided model, the probability that the ACO would be rewarded can be written as:

$$p_1 = \text{Prob}[ASR > T_n | \mu_4 = (1 - s)(\omega + A)] \quad (\text{A5})$$

Using the payment rules described above, the expected ACO payment can be written as:

$$p_1 \cdot E[ASR | ASR > T_n] \cdot (\omega + A) \cdot n \cdot \theta \quad (\text{A6})$$

where θ is the share of savings retained by the ACO (and $1 - \theta$ is the share retained by CMS).

Similar formulas can be written for the two-sided model. The probability of reward can be written as:

$$p_2 = \text{Prob}[ASR > 0.02 | \mu_4 = (1 - s)(\omega + A)] \quad (\text{A7})$$

In the two-sided model, there is also a probability that the ACO would have to pay a penalty, which can be written as:

$$q_2 = \text{Prob}[ASR < -0.02 | \mu_4 = (1 - s)(\omega + A)] \quad (\text{A8})$$

The expected ACO payment in the two-sided model (which incorporates the possibility of both ACO rewards and penalties) can be written as:

$$p_2 \cdot E[ASR | ASR > 0.02] \cdot (\omega + A) \cdot n \cdot \theta_S + q_2 \cdot E[ASR | ASR < -0.02] \cdot (\omega + A) \cdot n \cdot \theta_L \quad (\text{A9})$$

where θ_S is the share of savings retained by the ACO and θ_L is the share of losses that would be paid by the ACO in the form of a financial penalty.

Calculation of formulas A5–A9 is very complex. In contrast, the decision rules for statistical inference in the MSSP ACO payment formulas are much simpler. Simplicity is clearly an important element of the proposed formulas because it makes them transparent to all participants and allows CMS to calculate ACO payments in a timely way. In the next section of this Technical Appendix, we outline the decision rules for statistical inference that are embedded in the MSSP and show how our modeling framework can be made to conform to these rules with additional simplifying assumptions.

Statistical inference in the MSSP

Statistical inference in the MSSP is driven by the MSR threshold T_n . In the one-sided model, CMS specifies a “sliding scale confidence interval (CI) based on the number of assigned beneficiaries” (CMS, 2011). (In other words, the MSR represents the limit of a one-sided CI.) CIs are set at 90% for ACOs with 5,000 patients (the minimum allowable), 95% for ACOs with 20,000 patients, and 99% for ACOs with 50,000 beneficiaries. These CIs translate into the MSRs established by CMS (see Exhibit 1 in the main text). After setting these “anchor” MSRs, other MSRs are determined by linear interpolation for all other ACO sizes less than 60,000. For ACOs with 60,000 patients or more, the MSR is set at 2%.

Using our statistical model above, we are able to replicate exactly the three anchor MSRs set by CMS if we add some (admittedly very strong) simplifying assumptions:

1. Projected absolute amount of growth in per capita Medicare spending (A) is not subject to random variation.
2. $u_t = 0$ (i.e., no ACO random effects)
3. $\sum_{i=1}^n v_{it} = 0$ for $t=1, 2, 3$ (i.e., patient-level deviations from the mean cancel each other perfectly in the baseline years)
4. $\gamma = 0$ (i.e., deviation from expected spending is not correlated within the same patient over time).

These assumptions are similar to those made by Pope and Kautter (2011) in their more general analysis of potential ACO shared savings formulas.² (It should be noted that assumption 3, in particular, is quite strong but could be formulated differently, to the same effect, if all analyses are conditioned on the baseline values of v_{it})

Under these assumptions, $ASR = 1 - (\mu_4 + \bar{v}_4)/(\omega + A)$ where $\bar{v}_4 = \sum_{i=1}^n v_{i,4}/n$. Applying the Central Limit Theorem to \bar{v}_4 , ASR is normally distributed with mean $1 - \mu_4/(\omega + A)$ and variance

$$V(ASR) = (\omega + A)^{-2} V(\sum_{i=1}^n v_{i,4}/n) = \sigma_v^2/[n(\omega + A)^2] \quad (A10)$$

With these assumptions we calculate the probability that an ACO, which produces no real savings, will be rewarded inappropriately. In this case, ASR is normally distributed with mean equal to 0 [since $\mu_4 = (\omega + A)$] and variance as given in Equation 10. In doing this calculation, it is useful to view the ACO reimbursement mechanism as a statistical hypothesis test where the

²More specifically, Pope and Kautter (2011) consider the general problem of a payer entering a shared savings contract with a group of providers whose cost performance cannot be monitored perfectly, an issue known in the economics literature as the principle-agent problem. Their model is similar to ours in that they use the Central Limit Theorem as the basis for constructing a hypothesis test that would allow the payer to distinguish true savings from apparent savings that are driven by normal variation.

null hypothesis is that the ACO produces no savings. If the null hypothesis is true, there is a chance that the ACO will meet the MSR threshold due to random chance alone. This is known as the probability of Type I error and can be written as:

$$\text{Prob}[\text{ASR} > T_n | \mu_4 = (\omega + A)] \quad (\text{A11})$$

Exhibit 2 in the main text shows diagrammatically the relationship between T_n , the probability of Type I error, and random variability in ASR under the null hypothesis. Under CMS's proposed rules, T_n is set so that the probability of Type I error, which is the area to the right of T_n in Exhibit 2, equals 0.1 when $n=5,000$. This probability equals 0.05 when $n=20,000$ and it equals 0.01 when $n=50,000$.

To complete the calculation, we require an estimate of $\sigma_v^2/(\omega + A)^2$. This quantity reduces to $\frac{\sigma_v^2}{\mu_4^2}$ under the null hypothesis and can be viewed as the square of the coefficient of variation (CV) in patient expenditures within the ACO. Thus, we write the variance of ASR as $V(\text{ASR}) = \frac{1}{n} CV^2$. Using basic principles of statistical hypothesis testing, the null hypothesis is rejected if $(\frac{1}{\sqrt{n}} CV) > Z^*$, where Z^* is the critical value of the standard normal distribution. The rejection rule can be expressed alternatively as

$$\text{ASR} > (\frac{1}{\sqrt{n}} CV) Z^* = T_n \quad (\text{A12})$$

where T_n is the MSR threshold defined in the MSSP.

As described above, MSRs are based on one-sided confidence intervals corresponding to significance tests at the 10%, 5%, and 1% levels. These CIs are associated with the critical Z^* values in Exhibit A1. If CV is set at 2.15, then Inequality A12 produces the MSRs set by CMS as shown in Exhibit A1. (Or alternatively, one can use the parameters set by the MSSP and solve for CV in Inequality A12 as $CV = \sqrt{n}(Z^*/T_n)$ to derive $CV=2.15$.)

Exhibit A1. Derivation of minimum savings rates in proposed ACO rules.

ACO enrollment (n)	Confidence level for one-sided confidence interval	Critical value from standard normal distribution (Z^*)	Coefficient of variation (CV)	MSR threshold (T_n)
5,000	90%	1.28	2.15	0.039
20,000	95%	1.65	2.15	0.025
50,000	99%	2.33	2.15	0.022

SOURCE: Authors calculations derived from MSR thresholds in the Medicare Program.

We note as well that our estimate of the coefficient of variation is the same order of magnitude as that reported by Pope and Kautter (2011) and is in the middle of the range provided by other

studies that examine variability of Medicare spending (Ash et al., 2000; Counsell, Callahan, Tu, Stump, & Arling, 2009; Pope et al., 2000).³

In the next section of the Technical Appendix, we carry all of these assumptions forward to assess the statistical properties of the proposed ACO reimbursement formulas under varying scenarios of ACO performance. In the main text, we discuss how this assessment might change if the simplifying assumptions were relaxed.

Scenario analysis

We begin our scenario analysis by considering the situation where the true underlying savings rate for the ACO is zero. Then we calculate the expected financial liability for CMS (i.e., payment to the ACO) due solely to normal variation for a variety of ACO sizes (n) in light of the simplifying assumptions made above. To do so, we use Equation A6 for the one-sided model and Equation A9 for the two-sided model.

Next we consider eight scenarios of ACO performance to analyze the statistical properties of the proposed reimbursement formulas under the simplifying assumptions. In the first scenario, the ACO does not produce any savings relative to the updated benchmark [i.e. $(\omega + A)$]. Under the remaining seven scenarios, the ACO reduces per capita healthcare expenditures by 2%–10% relative to the updated benchmark. For both the one-sided and two-sided models, we calculate the probability that the ACO will be rewarded and the expected size of the reward, under each scenario using Equations A5–A9 for different ACO sizes (n). For the two-sided model, we also calculate the probability that the ACO would have to pay a penalty. In the two-sided model, the possibility of a negative reward (i.e., penalty) is factored into the expected income calculation as shown in Equation A9. All probability calculations are based on the probability density formula for the normal distribution, while the expected value calculations are based on properties of the truncated normal distribution as described in Greene (1998).⁴⁵

³Pope and Kautter (2011) report a CV estimate of 1.7, which they derived from the Medicare 5% claims and enrollment files. Their estimate is slightly lower than ours because they truncate expenditures at the 99th percentile. Other estimates using data from clinical trials or samples of paid Medicare claims generate CV estimates ranging from 1.32 to 2.60 (Ash et al., 2000; Counsell et al., 2009; Pope et al., 2000).

⁴As shown above, ASR is normally distributed with mean $1 - \mu_4/(\omega + A)$. In our scenario analysis, we assume that $\mu_4 = (1 - s)(\omega + A)$. Thus, the mean of ASR is s , which is the savings rate for the relevant scenario. Based on the variance of the ASR , which we also derived above, we find that the standard deviation for ASR is $\sigma = 2.15/\sqrt{n}$. To determine the probability that the ASR exceeds the relevant MSR threshold T_n for a given scenario (and ACO size n), we use standard notation in defining $\Phi(x)$ as the cumulative distribution function for the standard normal distribution (with mean of 0 and variance equal to 1). Thus, the probability that the ASR exceeds a threshold T_n is given by $1 - \Phi((T_n - s)/\sigma)$. To find the probability that the ASR is less than a given threshold (as in Equation A8), we calculate $\Phi((T_n - s)/\sigma)$ where T_n is set equal to -0.02 .

In our calculations, the variable s takes on the value 0 to 0.1 depending on the scenario considered. We assume that ACOs meet the quality and other standards set forth in the proposed rules making them eligible for the maximum proportion of any savings generated (and the minimum proportion of any losses) as shown in Exhibit 1 in the main text. We set the updated benchmark level of spending ($\omega + A$) equal to \$11,762, which is the average per capita level of Medicare spending in 2010 (The Boards of Trustees of the Federal Hospital Insurance and Federal Supplementary Medical Insurance Trust Funds, 2011). Our parameter assumptions are summarized in Exhibit A2. The results of our analysis are presented in the main text.

Exhibit A2. Values of key parameters in ACO scenario analysis.

Parameter	Value	Source
Reduction in average healthcare expenditure by the ACO (s)	0–0.1	Assumptions made for scenario analysis
Share of savings retained by the ACO under the one-sided model (θ)	0.5	Maximum allowable under proposed ACO rules ¹
Share of savings retained by the ACO under the two-sided model (θ_S)	0.6	Maximum allowable under proposed ACO rules ¹
Share of losses to be paid by the ACO under the two-sided model (θ_L)	0.4	1- maximum savings rate allowable under proposed ACO rules ¹
Baseline level of per capita healthcare expenditure ($\omega + A$)	\$11,762	Medicare Trustees, 2011

¹Assumes the ACO meets all quality and other performance standards set forth in the proposed rules.

⁵To calculate the expected value of the ASR conditional on the ASR exceeding the relevant MSR threshold, we use formulas for the truncated normal distribution derived in Greene (1998). Thus,

$E[ASR|ASR > T_n; \text{savings rate} = s] = s + \sigma \frac{\varphi((T_n - s)/\sigma)}{1 - \Phi((T_n - s)/\sigma)}$, where $\varphi(x)$ is the probability density function and $\Phi(x)$ is the cumulative distribution function for the standard normal distribution (with mean of 0 and variance equal to 1). The formula for expected value conditional on the ASR falling below -0.02 (as in Equation A8) is derived similarly.

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